Name: __KEY_____

EAS 426-08 MIDTERM EXAM

This exam is closed book and closed notes. It is worth 150 points; the value of each question is shown at the end of each question. At the end of the exam, you will find two pages of *potentially* useful equations.

1. The stress tensor at the Earth's surface, referred to a North-East-Down coordinate system, is given by the following matrix:

$$\sigma_{ij} = \begin{bmatrix} 100 & 40 & 20 \\ 40 & 60 & 10 \\ 20 & 10 & 30 \end{bmatrix}$$

   a. What are the normal and shear stresses on a plane that is parallel to the surface of the Earth? **[12 pts.]**

Because the coordinate system is NED, we are interested in the X_3 or the down axis. Because the first subscript indicates the plane perpendicular to that axis, the stresses on the surface will be σ_{31} = 20 (the shear stresses on the surface parallel to north), σ_{32} = 10 (the shear stresses on the surface parallel to east), and σ_{33} = 30 (the normal stress on the surface).

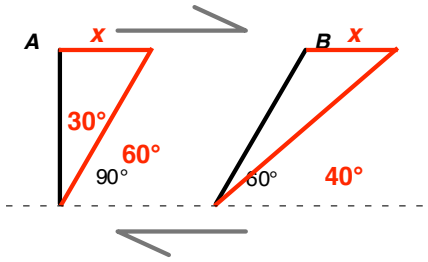
   b. What is the mean stress? **[10 pts.]**

$$\sigma_m = \frac{\sigma_{11} + \sigma_{22} + \sigma_{33}}{3} = \frac{100 + 60 + 30}{3} = \frac{190}{3} = 63.33$$

   c. Comment on this state of stress. Is it reasonable? Explain your answer. **[8 pts.]**

This state of stress is not particularly reasonable. The surface of the Earth is a "free surface" meaning that there are insignificant shear stresses on it, even when the wind is blowing hard. Thus, one of the principal stresses ought to be perpendicular to the surface. As shown in part (a), there are significant shear stresses in this problem.

2. The initial orientations of two lines in a shear zone are shown in the diagram, below:



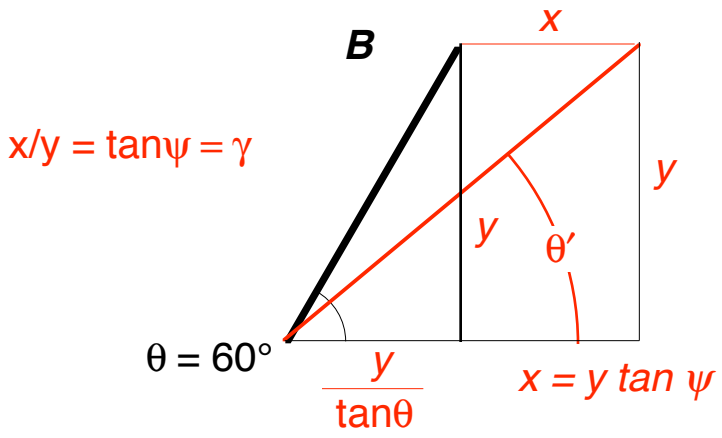
a. If the shear strain in the zone is 0.577, determine the final orientation of each line relative to the shear zone boundary. **[15 pts.]**

The shear strain, γ, is just the tangent of the angular shear, ψ. Therefore, the angular shear for this problem is:

$$\psi = \tan^{-1}(\gamma) = \tan^{-1}(0.577) = 30°$$

Line A is easy to calculate because its initial orientation is perpendicular to the shear zone, so it changes its orientation by 30° and has a final orientation relative to the shear zone of 60°. Now, one can just do a graphical solution by applying the same shear zone parallel translation, x, to the end of the line B as shown above.

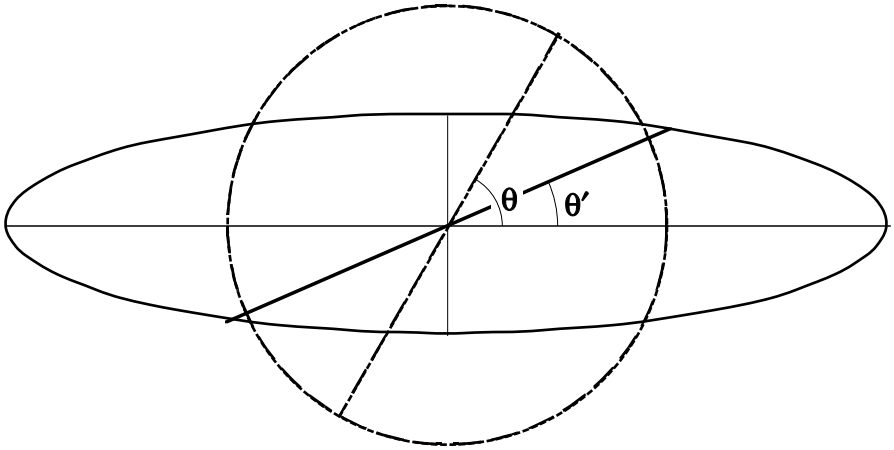
Alternatively, one can derive the trig equation to calculate the exact orientation:



$$\tan\theta' = \frac{y}{\frac{y}{\tan\theta} + y\tan\psi} = \frac{1}{\frac{1}{\tan\theta} + \tan\psi} = \frac{\tan\theta}{(1+\tan\psi\tan\theta)} = \frac{\tan 60}{(1+\tan 30\tan 60)} = 0.866$$

So, θ′ = 40.1°

b. Is the rotation that you calculated in part (a) the same as the rotation that one calculates when you calculate the rotation from the difference between θ and θ′ as shown in the figure, below? Explain your answer. **[10 pts.]**



No, the rotation is not the same. The rotation shown in the picture immediately above is an internal rotation with respect to the principal axes of strain. The rotations in part (a) of this problem are external rotations; they are not measured with respect to the principal axes of strain but with respect the external boundaries of the shear zone, i.e., a simple shear rotation.

3. Compare and contrast the following terms. Be specific, where possible, about the deformation mechanisms involved. Use sketches if you want.

   a. "Brittle" and "ductile". **[10 pts.]**

   Brittle means failure in a narrow zone of deformation, usually preceded only by linear elastic deformation. Ductile means broadly distributed deformation without loss of coherence.

   b. "Cataclastic" and "crystal plastic". **[10 pts.]**

   Cataclastic is brittle deformation accomplished by fracturing and rotating of individual grains or grain aggregates. Crystal plastic means large solid state changes in shape without fracture. Crystal plastic deformation is usually accomplished by dislocation glide and glimb.

   c. "Dislocation glide & climb" and "diffusion" **[10 pts.]**

   Dislocations are linear defects in crystals, the termination of an extra half plane, whereas diffusion relates to point defects. Dislocations move in crystallographic glide planes; when the encounter resistance to glide (strain hardening), if there are enough vacancies, the dislocations can climb to a different parallel glide plane and continue to move. This can only happen with diffusion.

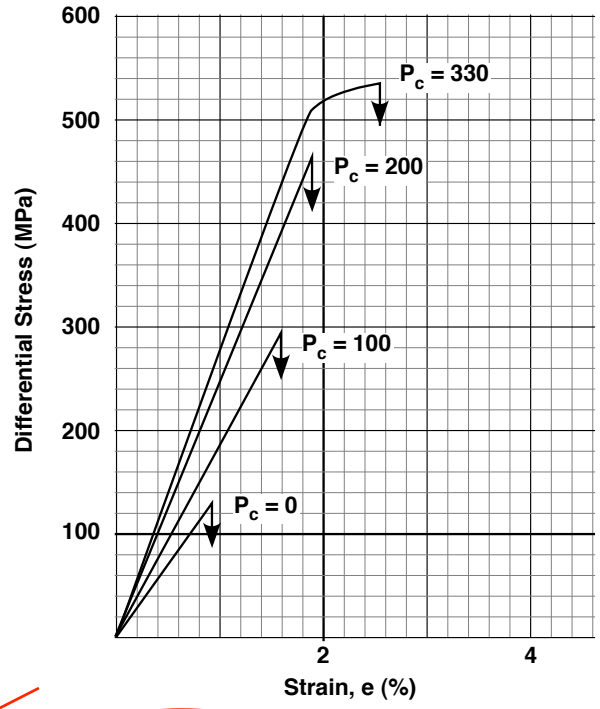
   d. "Plastic" and "viscoelastic" **[10 pts.]**

   Plastic deformation is permanent deformation that occurs without loss of cohesion once a threshold, or yield, stress is reached. Viscoelastic deformation is non-permanent deformation that accrues over time; the magnitude of the deformation is determined by the differential stress applied.

   e. "Diffraction" and "migration" **[10 pts.]**

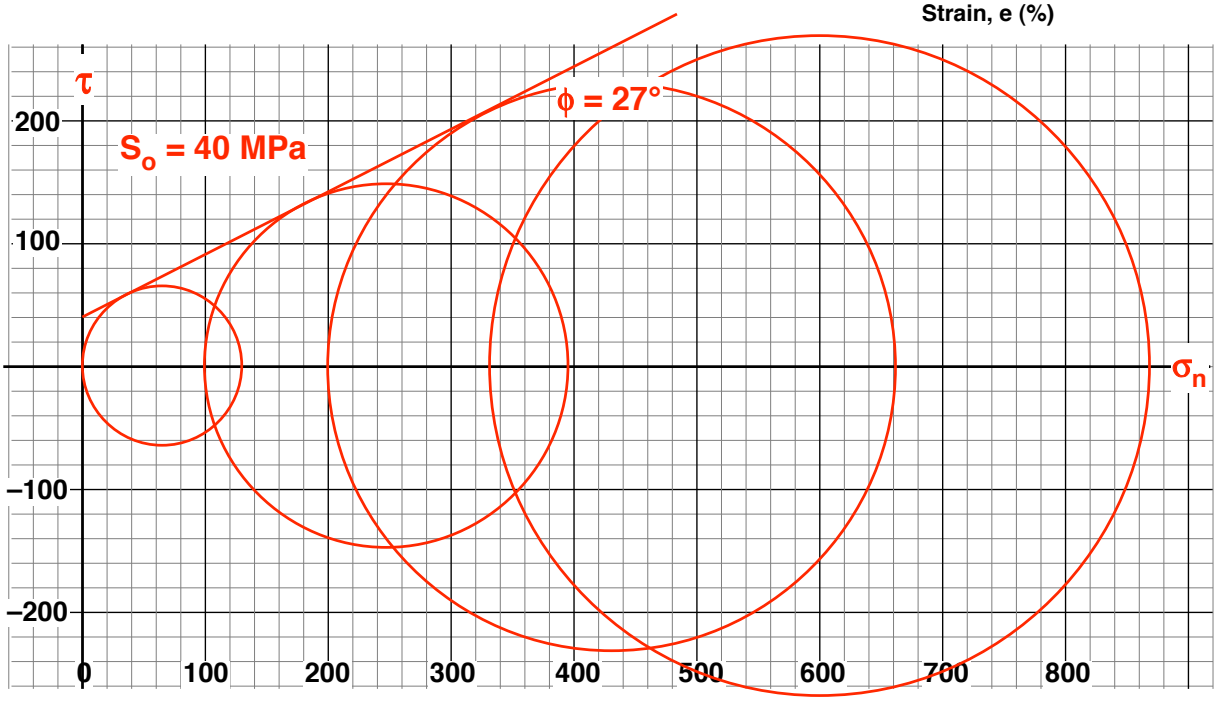
   A diffraction is the acoustic signature of a point source, which in seismic reflection profiling is anything smaller than the first fresnel zone. Migration is the correction applied to dipping reflectors to position them correctly in space. Migration will also collapse diffractions back to their point sources.

4. The plot at the right shows the results of a series of rock deformation experiments. P_c shows the confining pressure at failure for each of the experiments.



   a. Determine the failure envelop by plotting the experiments on the blank graph paper below. Be sure to label the axes and plot the cohesion, S_o. The units shown are MPa **[20 pts.]**

P_c is should plot as σ_3 on the Mohr's circle. The answer is plotted, below:



   b. At what angle(s) do the fractures form? How can you tell? **[10 pts.]**

The to poles to the fractures make an angle of

$$\theta = 45° + \frac{\phi}{2} = 45° + \frac{27°}{2} = 58.5°$$

c. Carefully explain what happens between the experiment with P_c = 200 Mpa and the experiment at P_c = 330 Pa. In your explanation, be sure to refer to different parts of the failure envelop and also describe the likely deformation mechanisms involved. **[15 pts.]**

Between these two experiments, we transition from Coulomb failure, where the differential stress at failure is linearly related to confining pressure, to Von Mises criterion plastic deformation where the failure is no longer related to confining pressure. The last experiment shows evidence of strain hardening plastic behavior.

Potentially Useful Equations

Note that not all of these equations are needed for the exam and that some of them have not, or will not, be covered in class.

$$\sigma_{\Delta T} = \frac{\alpha E \Delta T}{1-\upsilon}$$

$$\sigma_{ij} = \begin{vmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{vmatrix}$$

$$V_i = \frac{k_{ij}}{\eta}\left(\frac{dP_P}{dx_j}\right)$$

$$\sigma_n^* = \left(\frac{\sigma_1^* + \sigma_3^*}{2}\right) + \left(\frac{\sigma_1^* - \sigma_3^*}{2}\right)\cos 2\theta$$

$$\sigma_s = \left(\frac{\sigma_1^* - \sigma_3^*}{2}\right)\sin 2\theta$$

$$\lambda' = \left(\frac{\lambda_3' + \lambda_1'}{2}\right) - \left(\frac{\lambda_3' - \lambda_1'}{2}\right)\cos 2\theta'$$

$$\gamma' = \left(\frac{\lambda_3' - \lambda_1'}{2}\right)\sin 2\theta'$$

$$\tan\theta' = \tan\theta\frac{\sqrt{\lambda_3}}{\sqrt{\lambda_1}} = \tan\theta\frac{S_3}{S_1}$$

$$\sigma_s = S_o + \sigma_n^*\mu$$

$$\dot{\varepsilon} = C_o(\sigma_1 - \sigma_3)^n \exp\left(\frac{-Q}{RT}\right)$$

$$\dot{\varepsilon} = C_o(T)\frac{D(\sigma_1 - \sigma_3)}{d^n}$$

$$U = -\frac{C_1}{r} + \frac{C_2}{r^{12}}$$

$$P_{lith} = \int_0^z \rho g dz$$

$$\phi = \frac{V_f}{V_f + V_s}$$

$$\phi = \phi_o \exp(-az)$$

$$\Delta_v = \frac{V_{final} - V_{initial}}{V_{initial}}$$

$$e = \frac{l_f - l_i}{l_i}$$

$$e = \frac{\sin(\phi + \theta)}{\sin\phi} - 1$$

$$S = \frac{l_f}{l_i} = \sqrt{\lambda}$$

$$\lambda = S^2$$

$$\lambda' = \frac{1}{\lambda}$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos^2\theta = \left(\frac{1+\cos 2\theta}{2}\right)$$

$$\sin^2\theta = \left(\frac{1-\cos 2\theta}{2}\right)$$

$$U_i = U_{o_i} + E_{ij}dx_j$$

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} U_{o_1} \\ U_{o_2} \\ U_{o_3} \end{bmatrix} + \begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{21} & E_{22} & E_{23} \\ E_{31} & E_{32} & E_{33} \end{bmatrix} \begin{bmatrix} dx_1 \\ dx_2 \\ dx_3 \end{bmatrix}$$

$$\sigma_m^* = \frac{(\sigma_1 + \sigma_2 + \sigma_3 - 3P_f)}{3}$$

$$\sigma_1^* = C_o + K\sigma_3^*$$

$$K = \frac{1+\sin\phi}{1-\sin\phi} \;;\quad C_o = 2S_o\sqrt{K}$$

$$\sigma_\tau = 0.85\sigma_n^*$$

$$\sigma_\tau = 50\text{ MPa} + 0.6\sigma_n^*$$

$$(\alpha + \beta) = \frac{(1-\lambda_f)\mu_f + \beta}{(1-\lambda)k + 1}$$

R = 8.3144 x 10^{-3} kJ/mol °K
= 1.9872 x 10^{-3} kcal/mol °K

°K = °C + 273.16°

1 MPa = 106 kg/m s^2 = 10 bars

g = 9.8 m/s^2 = 980 cm/s^2

$$\cos\alpha = \cos(trend)\cos(plunge)$$
$$\cos\beta = \sin(trend)\cos(plunge)$$
$$\cos\gamma = \sin(plunge)$$
$$\cos\alpha = \sin(strike)\sin(dip)$$
$$\cos\beta = -\cos(strike)\sin(dip)$$
$$\cos\gamma = \cos(dip)$$

$$\tan 2\theta' = \frac{2}{\gamma}$$

$$p_i = \sigma_{ij} l_j$$
$$p_1 = \sigma_{11} l_1 + \sigma_{12} l_2 + \sigma_{13} l_3$$
$$p_2 = \sigma_{21} l_1 + \sigma_{22} l_2 + \sigma_{23} l_3$$
$$p_3 = \sigma_{31} l_1 + \sigma_{32} l_2 + \sigma_{33} l_3$$

$$L_d = 2\pi T_3 \sqrt{\frac{E}{6E_o}}$$

$$L_d = 2\pi T_3 \sqrt{\frac{\eta(S-1)}{6\eta_o(2S^2)}}$$

$$C \equiv \frac{1}{r}$$

$$C_G = C_{\max} C_{\min}$$

$$\beta = \theta - \phi + (180° - 2\gamma)$$

$$\phi = \tan^{-1}\left\{\frac{-\sin(\gamma-\theta)\left[\sin(2\gamma-\theta)-\sin\theta\right]}{\cos(\gamma-\theta)\left[\sin(2\gamma-\theta)-\sin\theta\right]-\sin\gamma}\right\}$$

$$\phi = \theta = \tan^{-1}\left\{\frac{\sin(2\gamma)}{2\cos^2(\gamma)-1}\right\}$$

$$\gamma = \tan\psi = 2\tan\left(\frac{\delta}{2}\right)$$

$$\gamma = \tan\psi \cong 0.0175(\delta)$$

$$\bar{s} = 2\bar{h}\tan\left(\frac{\delta}{2}\right)$$

$$\bar{s} \cong 0.0175\bar{h}(\delta)$$

$$\Sigma\Delta M = 0 = \Delta M_w + \Delta M_s + \Delta M_c + \Delta M_m + \Delta M_a$$

$$0 = \Delta(\rho_w h_w) + \Delta(\rho_s h_s) + \Delta(\rho_c h_c) + \Delta(\rho_m h_m) + \Delta(\rho_a h_a)$$

$$\Delta E = \Delta h_w + \Delta h_s + \Delta h_c + \Delta h_m + \Delta h_a$$